Question 1

(a) $\underset{\sim}{r} = (2t+t^3)\underset{\sim}{i} + (3-3t^2)\underset{\sim}{k}$

$$\underset{\sim}{v} = \frac{d\underset{\sim}{r}}{dt} = (2+3t^2)\underset{\sim}{i} - 6t\underset{\sim}{k}$$

$$\underset{\sim}{a} = \frac{d\underset{\sim}{v}}{dt} = 6t\underset{\sim}{i} - 6\underset{\sim}{k}$$

(i) At $t = 2$ sec.

$v_x = 2 + 3 \cdot 2^2 = 14$ $\quad v^2 = v_x^2 + v_z^2 = 340$

$v_z = -6 \cdot 2 = -12$

$K = \frac{1}{2}mv^2 = \frac{1}{2} \cdot 5 \cdot 340 = 850\ J$

(ii) $\underset{\sim}{p} = m\underset{\sim}{v} = 5(14\underset{\sim}{i} - 12\underset{\sim}{k}) = 70\underset{\sim}{i} - 60\underset{\sim}{k}$ $\quad$ kg m s^{-1}

(iii) $\underset{\sim}{F} = m\underset{\sim}{a} = 5(12\underset{\sim}{i} - 6\underset{\sim}{k}) = 60\underset{\sim}{i} - 30\underset{\sim}{k}$ $\quad$ Newton

(iv) $\underset{\sim}{L} = \underset{\sim}{r} \times \underset{\sim}{p}$

$= (12\underset{\sim}{i} - 9\underset{\sim}{k}) \times (70\underset{\sim}{i} - 60\underset{\sim}{k})$

$= -720\,\underset{\sim}{i} \times \underset{\sim}{k} - 630\,\underset{\sim}{k} \times \underset{\sim}{i} = 720\,\underset{\sim}{j} - 630\,\underset{\sim}{j} = 90\,\underset{\sim}{j}$ $\quad$ kg m^2 s^{-1}

(b) (i) Initial KE = Final PE + Work done against friction

$\frac{1}{2}mv_0^2 = mgh + fh$

$v_0^2 = 2gh + \frac{2f}{m}h = 2g(1 + f/mg)h$

$$h = \frac{v_0^2}{2g(1 + f/mg)}$$

(ii) Initial PE = Final KE + Work done against friction

$mgh = \frac{1}{2}mv^2 + fh$

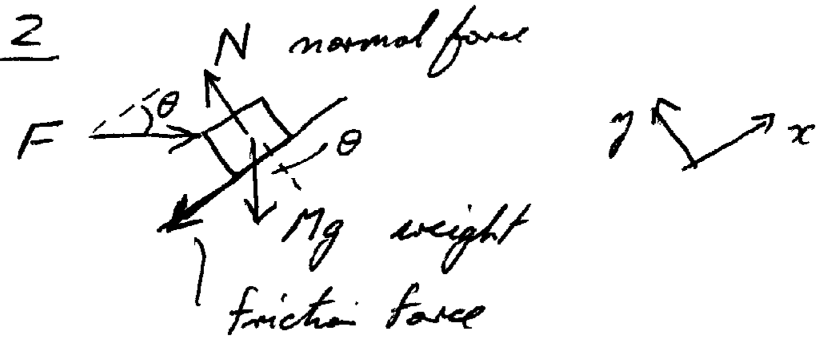
$v^2 = (2g - \frac{2f}{m})h$

$$= 2g\left(1 - \frac{f}{mg}\right) \cdot \frac{v_0^2}{2g(1 + f/mg)}$$

$$= \frac{mg - f}{mg + f}\, v_0^2$$

Question 2

(a)



(b) Forces parallel to plane

$$F_x = F\cos\theta - Mg\sin\theta - \mu_s N$$

Forces perpendicular to plane

$$F_y = N - F\sin\theta - Mg\cos\theta = 0$$

$$\therefore\ N = F\sin\theta + Mg\cos\theta$$

$$F_x = F\cos\theta - Mg\sin\theta - \mu_s F\sin\theta - \mu_s Mg\cos\theta$$
$$= F(\cos\theta - \mu_s\sin\theta) - Mg(\sin\theta + \mu_s\cos\theta)$$

The minimum force is

$$F^{(1)}_{min} = Mg\,\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}$$

(c) The friction force is directed <u>up</u> the plane

$$F_z = F\cos\theta - Mg\sin\theta + \mu_s(F\sin\theta + Mg\cos\theta)$$
$$= F(\cos\theta + \mu_s\sin\theta) - Mg(\sin\theta - \mu_s\cos\theta)$$

$$F^{(2)}_{min} = Mg\cdot\frac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta}$$

(d) The block will remain stationary if

$$F^{(2)}_{min} < F < F^{(1)}_{min}$$

Question 3

(a) Momentum and energy are conserved

(b) Let V and v be the final velocities of M and m

$$MU = MV + mv \quad \text{(conservation of momentum)}$$

$$\tfrac{1}{2}MU^2 = \tfrac{1}{2}MV^2 + \tfrac{1}{2}mv^2 \quad \text{(conservation of energy)}$$

(c) $$M(U-V) = mv \quad (1)$$

$$M(U^2-V^2) = mv^2$$

$$\therefore \quad U+V = v \qquad V = v - U \quad (3)$$

Subst (3) in (1) $\Rightarrow$

$$M(2U-v) = mv$$

$$2MU = (m+M)v$$

$$\boxed{\begin{array}{c}\text{Full marks if student states} \\ \text{Rel. velocity after collision} \\ = \text{rel. velocity before collision} \\ U = v - V\end{array}}$$

$$v = \frac{2MU}{m+M}$$

$$v_{max} = \frac{2MU}{M} = 2U \qquad (M \gg m)$$

(d) $$\frac{E_{block}}{E_{hammer}} = \frac{\tfrac{1}{2}mv^2}{\tfrac{1}{2}MU^2} = \frac{m}{M}\left(\frac{2M}{m+M}\right)^2$$

$$= \frac{4mM}{(m+M)^2}$$

$$\approx \frac{4m}{M} \qquad \text{if } m \ll M$$

Question 4

(a) $T = 24 \times 60 \times 60 = 8.64 \times 10^4$ sec.

$\omega = \frac{2\pi}{T} = 7.27 \times 10^{-5}$ rad. s^{-1}

(b)

(i) Between successive passages the Earth rotates through $\frac{1}{3} \times 360^\circ = 120^\circ$

The satellite will move through an angle of $360^\circ \pm 120^\circ$ for motion in the same or opposite direction

i.e. 480° or 240°

$\therefore \quad \omega = 4\omega_E$ or $2\omega_E$

(ii) Satellite equation

$$\frac{GM_E m}{r^2} = m\omega^2 r$$

$$\therefore r^3 = \frac{GM_E}{\omega^2}$$

$\omega = 4\omega_E$: $r = 1.68 \times 10^7$ m

$\omega = 2\omega_E$: $r = 2.66 \times 10^7$ m

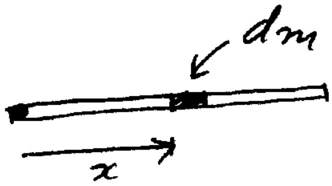
(iii) For a satellite

$U = -\frac{GM_E m}{r} \qquad K = -\frac{1}{2}U \qquad E = \frac{1}{2}U$

	U	K	E
$\omega = 4\omega_E$:	-2.38×10^9 J	1.19×10^9 J	-1.19×10^9 J
$\omega = 2\omega_E$:	-1.50×10^9 J	0.75×10^9 J	-0.75×10^9 J

Question 5

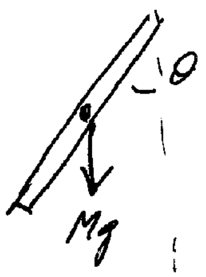
(a)



$$dm = \frac{M\,dx}{L}$$

$$I = \int x^2\,dm = \int_0^L x^2 \frac{M\,dx}{L} = \frac{M}{L}\left[\tfrac{1}{3}x^3\right]_0^L = \tfrac{1}{3}ML^2$$

(b) (i) The external force can be assumed to act at the centre of mass



$$\tau = Mg\cdot\frac{L}{2}\sin\theta = \tfrac{1}{2}MgL\sin\theta$$

(ii) $\tau = I\alpha$

$$\therefore \alpha = -\frac{\tfrac{1}{2}MgL}{\tfrac{1}{3}ML^2}\sin\theta = -\frac{3g}{2L}\sin\theta \quad (\text{"}-\text{" because restoring torque})$$

For small θ, $\sin\theta \simeq \theta$

$$\therefore \alpha = -\frac{3g}{2L}\theta = -\Omega^2\theta$$

This is the equation for angular SHM

(iii) Period $T = \dfrac{2\pi}{\Omega} = 2\pi\sqrt{\dfrac{2L}{3g}}$

Question 6

(a) Angular momentum is conserved

$$I_i \omega_i = I_f \omega_f$$

$$\therefore \quad \omega_f = \frac{I_i}{I_f}\omega_i$$

or $$T_f = \frac{I_f}{I_i} T_i \qquad \left(T = \frac{2\pi}{\omega}\right)$$

$$= \frac{\frac{2}{5} M R_f^2}{\frac{2}{5} M R_i^2} T_i = \left(\frac{R_f}{R_i}\right)^2 T_i$$

$$T_f = \left(\frac{3}{1 \times 10^4}\right)^2 \times 30 \times 24 \times 60 \times 60 = 0.23 \text{ sec.}$$

(b) Consider a mass m at the surface. This will not be "thrown off" provided the gravitational force is able to provide the necessary centripetal force

$$\frac{GMm}{R^2} = m\omega^2 R$$

$$\therefore \quad M = \frac{\omega^2 R^3}{G}$$

$$= \frac{4\pi^2 R^3}{T^2 G}$$

$$= \frac{4\pi^2 \cdot (3 \times 10^3)^3}{0.23^2 \times 6.673 \times 10^{-11}}$$

$$= 3.02 \times 10^{23} \text{ kg}$$